⟹ consider the following pts with ~~reg~~ negative class (2,1) (1,3) and positive class (6,3) points and find the support vectors. classify (7,6) is a well classified or not.

<u>solution:</u>

Hyperplane is $w^T x + b = 0$

for negative class $w^T x + b = -1$

$(2,1) \Rightarrow w^T(2,1) + b = -1 \longrightarrow (a)$

$(1,3) \Rightarrow w^T(1,3) + b = -1 \longrightarrow (b)$

for positive class $w^T x + b = +1$

$(6,3) \Rightarrow w^T(6,3) + b = 1 \longrightarrow (c)$

Let $w = (w_1, w_2)$ then

$2w_1 + w_2 + b = -1 \longrightarrow (1)$

$w_1 + 3w_2 + b = -1 \longrightarrow (2)$

$6w_1 + 3w_2 + b = 1 \longrightarrow (3)$

equa (2) − (1)

$(w_1 - 2w_1) + (3w_2 - w_2) + (b - b) = -1 - (+1)$

$\Rightarrow \quad -w_1 + 2w_2 + 0 = 0$

$w_1 = 2w_2 \longrightarrow (4)$

equa ② - ③

$(6w_1 - w_1) + (3w_2 - 3w_2) + (b - b) = 1 - (-1)$

$\rightarrow 5w_1 = 2 \Rightarrow w_1 = 2/5 \rightarrow$ ⑤

equating ④ & ⑤ equations

$$w_2 = \frac{w_1}{2} = \frac{2/5}{2} = 1/5$$

so $w = (2/5, 1/5) = (0.4, 0.2)$

place w values in equa ①

$$2w_1 + w_2 + b = -1$$

$$2(2/5) + 1/5 + b = -1$$

$$4/5 + 1/5 + b = -1$$

$$b = -1 - 4/5 - 1/5$$

$$b = \frac{-5 - 4 - 1}{5} = \frac{-10}{5} = -2$$

place w and b values in below equation

$w x + b = 0$ (hyper plane)

$0.4x_1 + 0.2x_2 - 2 = 0 \rightarrow$ ⑥

$w x + b = +1$ (+ve margin) upper margin

$$0.4x_1 + 0.2x_2 = 2 + 1$$

$0.4x_1 + 0.2x_2 = 3 \rightarrow$ ⑦

$w x + b = -1$ (-ve margin)(lower margin)

$$0.4x_1 + 0.2x_2 = 2 - 1$$

$0.4x_1 + 0.2x_2 = 1 \rightarrow$ ⑧

⇒ solving ⑥ $0.4x_1 + 0.2x_2 = 2$

if $x_1 = 0$ then $x_2 = \frac{2}{0.2}$

$x_2 = 10$

$(x_1, x_2) = (0, 10)$

Solving if $x_2 = 0$ then $x_1 = \frac{2}{0.4} = 5$

$x_1 = 5$

$(x_1, x_2) = (5, 0)$

⇒ Solving ⑦ if $x_1 = 0$ then $x_2 = \frac{3}{0.2} = 15$

$(x_1, x_2) = (0, 15)$

if $x_2 = 0$ then $x_1 = \frac{3}{0.4} = 7.5$

$(x_1, x_2) = (7.5, 0)$

⇒ Solving eq ⑧ $0.4x_1 + 0.2x_2 = 1$

if $x_1 = 0 \Rightarrow 0.2x_2 = 1 \Rightarrow x_2 = \frac{1}{0.2} = 5$

$(x_1, x_2) = (0, 5)$

if $x_2 = 0 \Rightarrow 0.4x_1 = 1 \Rightarrow x_1 = \frac{1}{0.4} = 2.5$

$(x_1, x_2) = (2.5, 0)$

Finding support vectors:

As we know from legranges formula

$$w = \sum_{i=1}^{n} \alpha_i y_i x_i$$

$$w = \alpha_1 (-1)(2,1) + \alpha_2 (-1)(1,3) + \alpha_3 (1)(6,3)$$

$$\Rightarrow (-2\alpha_1 - \alpha_2 + 6\alpha_3 \rightarrow (9),$$
$$-\alpha_1 - 3\alpha_2 + 3\alpha_3 \rightarrow (10))$$

similarly $b = \sum_{i=1}^{n} \alpha_i y_i = 0$

$$-\alpha_1 - \alpha_2 + \alpha_3 = 0$$

$$\alpha_3 = \alpha_1 + \alpha_2 \rightarrow (11)$$

substitute α_3 in (9) and (10)

$$-2\alpha_1 - \alpha_2 + 6\alpha_3 = 0.4$$

$$-2\alpha_1 - \alpha_2 + 6(\alpha_1 + \alpha_2) = 0.4$$

$$-2\alpha_1 - \alpha_2 + 6\alpha_1 + 6\alpha_2 = 0.4$$

$$4\alpha_1 + 5\alpha_2 = 0.4 \rightarrow (12)$$

substitute α_3 in (10)

$$-\alpha_1 - 3\alpha_2 + 3\alpha_3 = 0.2$$

$$-\alpha_1 - 3\alpha_2 + 3(\alpha_1 + \alpha_2) = 0.2$$

$$-\alpha_1 - 3\alpha_2 + 3\alpha_1 + 3\alpha_2 = 0.2$$

$$2\alpha_1 = 0.2$$

$$\alpha_1 = \frac{0.2}{2} = 0.1$$

substitute $\alpha_1 = 0.1$ in (12)

$$4\alpha_1 + 5\alpha_2 = 0.4$$

$$4\left(\frac{0.2}{2}\right) + 5\alpha_2 = 0.4$$

$$\alpha_2 = 0$$

Similarly after substituting

$$\alpha_3 = 0.1$$

Put $\alpha_1, \alpha_2, \alpha_3$ in equa (9)(10) to verify

$$w = -2\alpha_1 - \alpha_2 + 6\alpha_3 = 0.4$$

$$= -2(0.1) - 0 + 6(0.1)$$

$$= -0.2 + 0.6 = 0.4$$

Similarly

$$w = -\alpha_1 - 3\alpha_2 + 3\alpha_3 = 0.2$$

$$= -0.1 - 0 + 3(0.1)$$

$$= 0.2$$

so finally $w = (0.4, 0.2)$

so support vectors are

$\alpha_1 = 0.1$ (support vector (2,1))

$\alpha_2 = 0$ (It lies on margin line but it doesnot contribute to w)

$\alpha_3 = 0.1$ (support vector (6, 3))

(7, 6) keep in eq (6)

$\Rightarrow 0.4 x_1 + 0.2 x_2 - 2 = 0$

$\Rightarrow 0.4(7) + 0.2(6) - 2$

$\Rightarrow 2 \times 8 + 1 \times 2 - 2$

$\Rightarrow 4 - 2$

$\Rightarrow 2 > 0$

(7, 6) belongs to positive class